Course Description: A first introduction to abstract algebra through group theory, with an emphasis on concrete examples, and especially on geometric symmetry groups. The course will introduce basic notions (groups, subgroups, homomorphisms, quotients) and prove foundational results (Lagrange’s theorem, Cauchy’s theorem, orbit-counting techniques, the classification of finite abelian groups). Examples to be discussed include permutation groups, dihedral groups, matrix groups, and finite rotation groups, culminating in the classification of the wallpaper groups.

Course Prerequisite: 110.201 Linear Algebra or equivalent.

Course Category: This is an Introduction to Proofs course (IP) and may be taken as a first proof-based mathematics course. This course also satisfies a core requirement of the mathematics major.


Course Topics (chapters to be covered)

1. Symmetries of the tetrahedron
2. Group axioms
3. Number groups
4. Dihedral groups
5. Subgroups and generators
6. Permutations
7. Isomorphisms
8. Plato’s solids and Cayley’s theorem
9. Matrix groups
10. Products
11. Lagrange’s theorem
12. Partitions
13. Cauchy’s theorem
14. Conjugacy
15. Quotient groups
16. Homomorphisms
17. Actions, orbits, and stabilizers
18. Counting orbits
19. Finite rotation groups
20. Finitely generated abelian groups
21. Row and column operations
22. Automorphisms
23. Euclidean group
24. Lattices
25. Wallpaper patterns