GEOMETRIC ANALYSIS

Honoring Bernie Shiffman’s Service to the Department

• Jean-Pierre Demailly, University of Grenoble, *Hyperbolicity of general algebraic hypersurfaces of high degree*

  **Abstract:** A study of the geometry of Semple jet bundles, combined with recent ideas of Damian Brotbek, leads to a proof of the Kobayashi conjecture on the hyperbolicity of algebraic hypersurfaces: a general algebraic hypersurface of sufficiently high degree in complex projective space is Kobayashi hyperbolic. The same technique gives an alternative and unified proof of the hyperbolicity of certain hypersurfaces of low degree constructed by Bernie Shiffman and Mikhail Zaidenberg in 2001.

• Damien Gayet, University of Grenoble, *Percolation of random nodal lines*

  **Abstract:** Bargmann-Fock random functions on $\mathbb{C}^n$ are the universal local rescaled limits of random holomorphic sections of the $d$-th power of an ample line bundle over a projective manifold, when the degree $d$ grows to infinity. In these settings, the natural scale is $1/\sqrt{d}$ for the latter, and $1$ for the former. I will explain a large scale result for the real Bargmann-Fock random functions (rBF): for any given rectangle $R$ in $\mathbb{R}^2$, there exists a constant $c > 0$, such that for any large enough homothetical copy $nR$ of $R$, with probability at least $c$, there exists a connected component of the vanishing locus of a random function in rBF which crosses $nR$ in its length. This is a joint work with Vincent Beffara (Institut Fourier, Grenoble).

• Gang Liu, Northwestern University, *Gromov-Hausdorff limits of Kähler manifolds with Ricci curvature lower bound*

  **Abstract:** A fundamental result of Donaldson-Sun states that non-collapsed Gromov-Hausdorff limits of polarized Kähler manifolds, with 2-sided Ricci curvature bounds, are normal projective varieties. We extend their approach to the setting where only a lower bound for the Ricci curvature is assumed. More precisely, we show that non-collapsed Gromov-Hausdorff limits of polarized Kähler manifolds, with Ricci curvature bounded below, are normal projective varieties. In addition the metric singularities are precisely given by a countable union of analytic subvarieties. This is a joint work with Gabor Szekelyhidi.

• William Minicozzi, MIT, *Uniqueness of blow ups for geometric flow*

  **Abstract:** One of the fundamental questions in regularity theory is whether there is uniqueness of blow ups. I will talk about recent progress with Toby Colding on uniqueness for nonlinear elliptic and parabolic systems. Our results
imply regularity of the singular set for the system and partial regularity of the solutions.

- Junjiro Noguchi, University of Tokyo, *A big Picard theorem for semi-abelian varieties and torsion points*

  **Abstract:** There are a number of analogues between the value distribution theory and Diophantine geometry at the statement level by S. Lang, I. Shafarevich, P. Vojta, ... In the present talk we would like to discuss a more direct relation between them at the proof level.

  A big Picard theorem proved by the author 1981 says that if a holomorphic map from a punctured disk into a semi-abelian variety has an essential singularity at the puncture, then the Zariski closure of the map has a positive dimensional stabilizer.

  Using this theorem, we give a new proof to the so-called Manin-Mumford conjecture, which was proved by M. Raynaud in 1983 for abelian varieties and generalized by many authors, M. Hindry, E. Hrushovski, McQuillan, ..., Pila-Zannier.

  Here we prove that if $X$ is a subvariety of a semi-abelian variety defined over a number field, then the Zariski closure of the set of torsion points in $X$ is a union of finitely many translates of algebraic subgroups.

  It is interesting to observe that the link between the big Picard theorem and the above arithmetic problem is given by the theory of “$o$-minimal structure” in Logic.

- D. H. Phong, Columbia University, *Geometric Partial Differential Equations from M Theory*

  **Abstract:** Since the mid 1990’s, the leading candidate for a unified theory of all fundamental physical interactions has been M Theory. A full formulation of M Theory is still not available, and it is only understood through its limits in certain regimes, which are either one of five 10-dimensional string theories, or 11-dimensional supergravity. The equations for these theories are mathematically interesting in themselves, as they reflect, either directly or indirectly, the presence of supersymmetry. We discuss recent progresses and open problems about two of these theories, namely supersymmetric compactifications of the heterotic string and of 11-dimensional supergravity. This is based on joint work of the speaker with Sebastien Picard and Xiangwen Zhang, and with Teng Fei and Bin Guo.

- Yanir Rubinstein, University of Maryland, *Differential, algebraic, and convex geometry arising from asymptotic positivity*

  **Abstract:** A general theme in geometry is the classification of algebraic/differential geometric structures which satisfy a positivity property. I will describe an “asymptotic” version of this theme based on joint work with Cheltsov, Martinez-Garcia, and Zhang. On the algebraic side, we introduce the class of asymptotically log Fano varieties and state a classification theorem in dimension 2, generalizing the classical efforts of the 19th century Italian school. The novelty here is the use of a convex optimization theorem that reduce the asymptotic positivity to determining intersection properties of high-dimensional convex bodies.
On the differential side, I will give a conjectural picture for existence of singular Kähler-Einstein metrics and explain progress towards this conjecture making use of symmetry, log canonical thresholds, test configurations, and Fujita-Odaka’s basis type invariant. Time permitting, I will also touch on relations to singular Kähler-Ricci solitons, mention some conjectures and results about the ‘small angle limit’ when the angle tends to zero, and tie this picture to non-compact Calabi-Yau fibrations, steady Ricci solitons, and recent work of Liu on wall-crossings in moduli space.

• Yum-Tong Siu, Harvard University, *Splitting of unstable 2-bundles over the complex projective 6-space*

  **Abstract:** The conjecture that every unstable 2-bundle over the complex projective 4-space splits has been open since the 1970s. A few years ago a preprint in arXiv.org introduced the method of holomorphic vector fields and Barth-Lefschetz techniques for the splitting of unstable 2-bundles over the complex projective 6-space, but one important case was not handled in it. Here we refine the method of holomorphic vector fields and Barth-Lefschetz techniques to handle the remaining important case and present a proof of the splitting of unstable 2-bundles over the complex projective 6-space.

• Jean-Yves Welschinger, University Lyon, *Tilings, packings and expected Betti numbers in simplicial complexes*

  **Abstract:** I will explain how to bound from above the expected Betti numbers of a random subcomplex in a simplicial complex. I will then explain how packings of simplices make it possible to improve these upper bounds and finally discuss a notion of tilings which makes it possible to produce such packings. This is a joint work with Nermin Salepci.

• Shing-Tung Yau, Harvard University, *HSM/SYZ mirror symmetry: Recent progress and going forward*

• Steve Zelditch, Northwestern University, *From random zero sets to random Kähler metrics*

  **Abstract:** Bernie and I wrote 20 articles together about the zero sets of a $k$-tuple of independent random holomorphic sections of positive Hermitian line bundles over a Kähler manifold of dimension $m \geq k$. The current of integration over the zeros defines a singular Kähler metric. In recent years, there has been a lot of interest in random Kähler metrics of more general kinds. My talk is an overview of my joint work with Bernie and of some of the more recent results on random Kähler metrics.