

George R. Kempf

I met George in 1970 when he burst on the algebraic geometry scene with a spectacular PhD thesis. His thesis gave a wonderful analysis of the singularities of the subvarieties W_r of the Jacobian of a curve C obtained by adding the curve to itself r times inside its Jacobian. This was one of the major themes that he pursued throughout his career: understanding the interaction of a curve with its Jacobian and especially to the map from the r -fold symmetric product of the curve to the Jacobian. In his thesis he gave a determinantal representation both of W_r and of its tangent cone at all its singular points, which gives you a complete understanding of the nature of these singularities. A major focus of his later work in this area were the Picard bundles: the vector bundles on the Jacobian whose projectivization are r -fold symmetric powers of C , for $r \geq 2g-1$. He unwound many of the mysteries of these bundles.

As George's research evolved, our work became closely intertwined in multiple ways. In particular, he worked on invariant theory and on abelian varieties, esp. linear systems on abelian varieties. Maybe his result in invariant theory which I loved the most concerns the orbits in a representation of a reductive group which are 'unstable', i.e. have 0 in their closure. He gave a beautiful construction of one canonical worst subgroup \mathbf{G}_m in G carrying the point to 0. I had looked for this in awkward ways and found it in some cases, but he saw what was really going on. This result had many Corollaries and completed the program in Geometric Invariant Theory in the best possible way. Later on, he studied extensively the singularities of orbit spaces, showing in many cases that they had only rational singularities and he also studied the effective construction of rings of invariants, and thus of orbit spaces.

Perhaps the area in which we were closest was his work on linear systems and the equations defining abelian varieties. I wrote three papers on this in 1966-67, much inspired by hearing Igusa's lectures on theta functions. But I used to joke that George was the only one in the world who actually read these papers. Again, he went deeper than I with more persistence and the deft touch by which I always recognized his work. He kept finding better and more satisfying reasons why abelian varieties are so wonderful. For example, there was his theorem that their homogeneous coordinate rings A were, in his terminology, exactly that: 'wonderful'. He defined 'wonderful' to mean that all the modules $\mathrm{Tor}_i^A(k, k)$ are purely of degree i . This turns out to be the secret cohomological key to answer many questions. Another unexpected and lovely result was the one he dedicated to me for my 50th birthday: that multiplication gives an isomorphism between the tensor product of the vector space of rank 2 theta functions, generically twisted, and the vector space of rank 4 theta functions.

One of the things that distinguished his work was the total mastery with which he used higher cohomology. A paper which, I believe, every new student of algebraic geometry should read, is his elementary proof of the Riemann-Roch theorem on curves: 'Algebraic Curves' in Crelle, 1977. That such an old result could be treated with new insight was the work of a master.

I won't discuss his work on the cohomology of homogeneous spaces or the representation theory of algebraic groups, which others know much better than I. Instead, I want to conclude by saying that this love of the simple and satisfying elegance which can be found in these abstract fields brought George and I together. One feels that, given the disease with which he struggled, this mathematics was a constant stable light to which he returned, that centered him when other things failed. We miss the light he shed for us.